

Evaluation of the Transient Countercurrent Double Pipe Heat Exchanger with Variable Specific Heat Coefficient

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Abstract

In this paper, the unsteady state heat transfer from countercurrent double pipe heat exchanger is presented. A finite difference technique is used to solve the unsteady countercurrent double pipe heat exchanger differential equations. The variable specific heat coefficient with temperature is taken in account during the present analysis. Three types of effectiveness are discussed here because the change of the internal energy for the cold, hot fluids and also for the pipe wall material is taken in account. The entropy generation number for the case study is studied and the variation of the entropy generation number with effectiveness and other parameter is examined. The effect of the time, N_c , N_h , E_h , C_r on the cold & hot fluid effectiveness are examined, the increasing of the N_c & N_h values will increase the hot fluid effectiveness & in same time decrease the cold fluid effectiveness & also the increasing of the C_r will increase the hot fluid effectiveness and the entropy generation number. Our analysis is considering hot fluid (that we need to cool it) flow inside the tube & the cold fluid flows inside the annulus area.

الخلاصة

في هذه الدراسة ، لقد تم دراسة انتقال الحرارة الغير مستقرة لمبادل حراري متعاكس الجريان ذو أنابيب متداخلة (countercurrent double pipe). لقد تم استخدام طريقة الفروقات المحددة (Finite Difference) لحل المعادلات التفاضلية للمبادل الحراري المتعاكس. إن تغير معامل الحرارة الخاص (Specific Heat Coefficient) مع درجة الحرارة أؤخذ في نظر الاعتبار أثناء التحليل الحالي. بسبب أخذ بنظر الاعتبار تغيير الطاقة الداخلية للمائع الحار والبارد وأيضاً لمادة الأنبوب ، لذلك تم مناقشة ثلاثة من أنواع الفاعلية (Effectiveness). إن تأثير الوقت، N_c ، N_h ، E_h ، C_r على المائع البارد والحار تم دراسة أيضاً. إن زيادة قيمة (N_c, N_h) يؤدي إلى زيادة فاعلية للمائع الساخن وتقليل فاعلية المائع البارد، وإن زيادة قيمة (C_r) يؤدي إلى زيادة فاعلية للمائع الساخن. خلال التحليل، لقد تم اعتبار المائع الحار (الذي نحتاج لتبريده) يتدفق داخل الأنبوب والمائع البارد يتدفق داخل المنطقة المحصورة بين الأنابيب.

NOMENCLATURES

Q	The heat transfer rate (W)	P	Perimeter (m)
\dot{m}	Mass flow rate (Kg/Sec)	L	Length of the heat exchanger (m)
C_p	Constant pressure mass heat capacity (KJ/Kg.K)	D	Diameter (m)
T	Temperature (°C)	r	Radius (m)
x	Longitudinal distance in x-direction (m)	k	Thermal conductivity (W/m.K)
V	Volume (m ³)	t_w	Thickness of the inlet pipe (m)
t	Time (sec)	Pr	Prandtl number
h	Heat transfer coefficient (W/m ² .K)	Re	Reynolds number
A	Area (m ²)	f	Darcy-Weisbach friction coefficient

Greek

β	Constant for heat capacity coefficient (1/K)	α	Dimensionless constant for heat capacity coefficient
ρ	Density (Kg/m ³)	Δ	Change during element
μ	Dynamic viscosity (Pa.Sec)	ν	Kinematics viscosity (m ² /Sec)
ε	Heat exchanger effectiveness		

Dimensionless Quantities

$\theta_{c,h,w}(x,t)$	$(T_{c,h,w}(x,t) - T_{c,i}) / (T_{h,i} - T_{c,i})$	X'	x/L
$C_{p,c,h,w}$	$C_{p,oc,h,w} (1 + \alpha \theta_{c,h,w})$	τ	t/τ_0

$$N_{c,h} = \frac{h_{c,h} A_{c,h}}{\dot{m}_{c,h} C_{p_{oc,h}}} \quad \tau_o = \frac{\rho_c V_c C_{p_{oc}}}{\dot{m}_c C_{p_{oc}}}$$

$$c_r = \frac{\dot{m}_h C_{p_{oh}}}{\dot{m}_c C_{p_{oc}}} \quad E_{c,h,w} = \frac{\rho_{c,h,w} V_{c,h,w} C_{p_{oc,h,w}}}{\rho_c V_c C_{p_{oc}}}$$

Subscript

c	Cold fluid	w	Pipe wall temperature
c,i	Inlet cold fluid	in	Inlet
c,o	Outlet cold fluid	out	Outlet
h	hot fluid	m	Distance location in X-direction
h,i	Inlet hot fluid	max	Maximum
h,o	Outlet hot fluid	a	Instantaneous actual
h,m	Hot to pipe wall	c,m	Cold to pipe wall

Superscript

'	Dimensionless distance	n	Time increasing
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Introduction

Heat exchangers, as the name implies, transfer heat from one substance to another. A heat exchanger is a heat transfer device that exchanges heat between two or more process fluids. Heat exchangers have widespread industrial and domestic applications and in the process industry for the recovery of the heat. They are an essential component in thermal power system, refrigeration system, and other cooling system. In all these systems, heat is transfer from one fluid to other. The countercurrent heat exchanger is most favorite because it is gives maximum rate of heat transfer for a given surface area

In many thermal equipments design are based on the steady state in calculation of the characteristic variable. Many applications require knowledge of the transient behavior of the thermal devices. In fact, it is necessary to explore the unsteady state of thermal properties when the real time control, state computation, optimization, and rational use of energy are investigated. In additional, the unsteady state gives more details and information than steady state & also gives indication to validity of the steady state assumption.

The steady state heat exchanger is well defined & discussed in many literatures, [1,2,3,4,5].

The steady state solution are commonly divided in to two methods, the first method named “Log Mean Temperature Difference (LMTD)”, this method is applied when the inlet & outlet temperatures are knowing. The second method is named “Effectiveness- Number of Transfer Unit (ϵ –NTU)”, this method applied when the outlet temperatures of the fluids are unknowing, this method is based on calculation of maximum heat transfer rate & number of transfer units. These methods are widely discussed in many literature & textbooks, [1,2,3,4,5,6].

Bagul,[7], indicate anew steady state formulation for countercurrent double pipe heat exchanger with constant parameter experimentally.

In our study , we analyzed countercurrent heat exchanger which used to cool a hot fluid (flow inside tube) by a cooled fluid (flow inside the annulus area). The finite difference techniques was used to solve the unsteady state countercurrent double pipe heat exchanger differential equations, The variable specific heat coefficient with temperature was taken in account during the present analysis.

Analytical Analysis

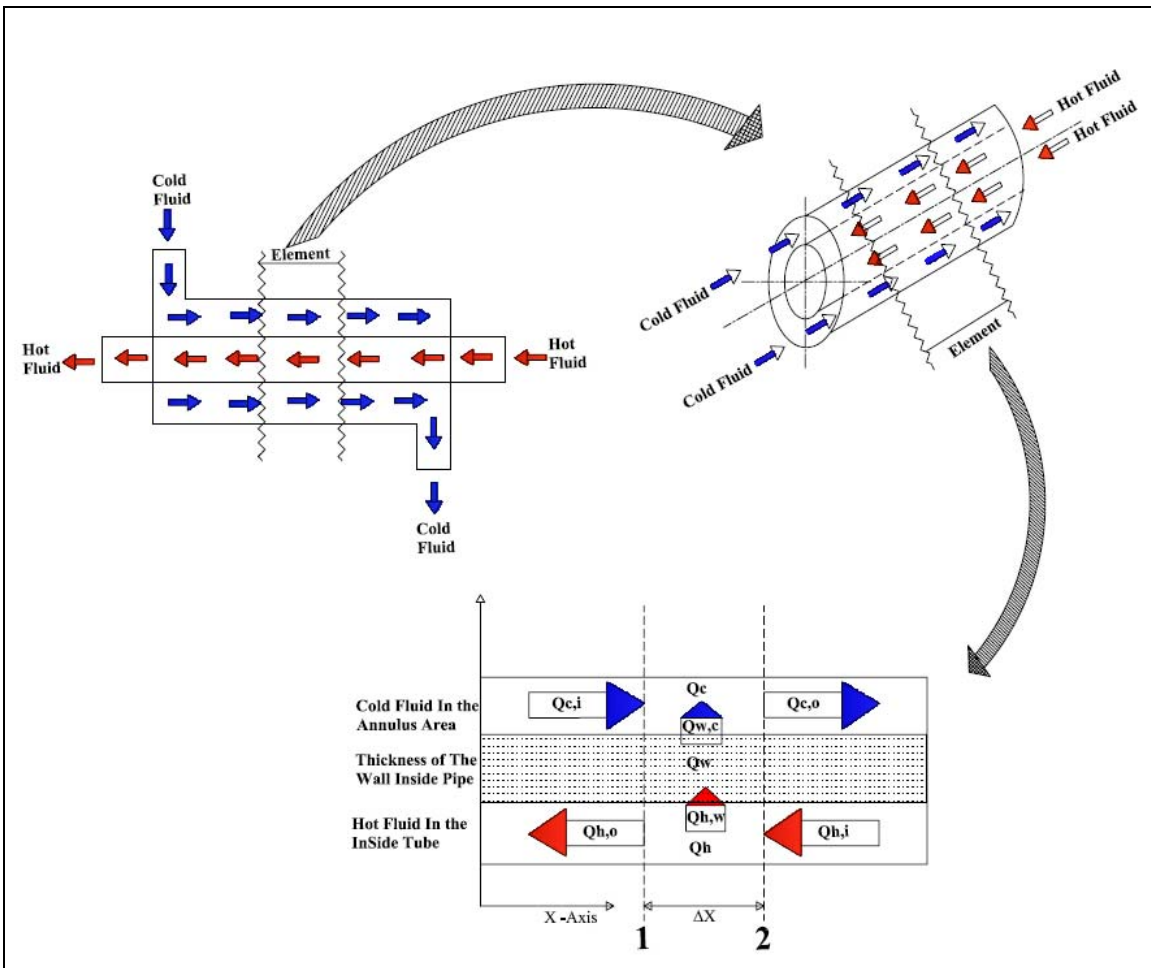
The assumptions that used in the theoretical analysis of countercurrent double pipe heat exchanger are following:

- 1- There is no phase change for the hot & cold fluids during the heat exchanger pipes.
- 2- Hot & cold fluids are incompressible & turbulent in flow.

- 3- The axial heat conduction inside the tubes wall is negligible (the convection resistance is too high compared width conduction resistance).
- 4- The hot, cold fluids & the pipe wall temperature subject to unsteady state behavior.
- 5- The specific heat coefficient is variable with temperature.
- 6- The hot fluid flow inside the inner pipe & the cold fluid flow inside the annulus area between pipes.

In order to derive the governor differential equations, the heat exchanger is subdivided in to many elements volumes of length (Δx) as shown in fig (1). The hot fluid flows through the element volume transfer heat to the wall by convection , resulting in reduction in the outlet enthalpy and internal stored thermal energy.

Energy balance applied to the differential volume of the hot fluid leads to the first equation in the system. Similar energy balance is applied for the cold fluid & separation cylinder wall. The energy balance for the hot, cold fuel & for separation wall were done as following:



Fig(1): Schematic of Double Pipe Heat Exchanger

For Cold Fluid

At section 1 $Q_{c,i} = \dot{m}_c C_{p_c} T_c$ (1)

Where $C_{p_c} = C_{p_{oc}} (1 + \beta T_c)$ (2)

$$\text{At section 2 } Q_{c,o} = Q_{c,i} + \frac{\partial Q_{c,i}}{\partial x} \Delta x \dots\dots\dots (3)$$

$$\text{The change of internal energy in cold fluid } Q_c = \rho_c \Delta V_c C p_c \frac{\partial T_c}{\partial t} \dots\dots\dots (4)$$

$$\text{Where } \Delta V_c = V_c \frac{\Delta x}{L} \dots\dots\dots (5)$$

$$\text{Then } Q_c = \rho_c V_c \frac{\Delta x}{L} C p_c \frac{\partial T_c}{\partial t} \dots\dots\dots (6)$$

For Hot Fluid

$$\text{At section 2; } Q_{h,i} = \dot{m}_h C p_h T_h \dots\dots\dots (7)$$

$$\text{Where } C p_h = C p_{oh} (1 + \beta T_h) \dots\dots\dots (8)$$

$$\text{At section 1; } Q_{h,o} = Q_{h,i} + \frac{\partial Q_{h,i}}{\partial x} \Delta x \dots\dots\dots (9)$$

The change of internal energy in hot fluid;

$$Q_h = -\rho_h \Delta V_h C p_h \frac{\partial T_h}{\partial t} = -\rho_h V_h \frac{\Delta x}{L} C p_h \frac{\partial T_h}{\partial t} \dots\dots\dots (10)$$

For Pipe Material

The change of internal energy in pipe material;

$$Q_w = \rho_w \Delta V_w C p_w \frac{\partial T_w}{\partial t} = \rho_w V_w \frac{\Delta x}{L} C p_w \frac{\partial T_w}{\partial t} \dots\dots\dots (11)$$

$$\text{Where } C p_w = C p_{ow} (1 + \beta T_w) \dots\dots\dots (12)$$

The heat removed from the hot fluid to the pipe wall;

$$dQ_{h,w} = h_h (T_h - T_w) dA_h \dots\dots\dots (13)$$

The heat removed from the pipe wall to the cold fluid;

$$dQ_{w,c} = h_c (T_w - T_c) dA_c \dots\dots\dots (14)$$

Where

$$dA_c = P_c \Delta x, dA_c = A_c \frac{\Delta x}{L}, A_c = P_c L \text{ \& } P_c = \pi D \dots\dots\dots (15)$$

$$dA_h = P_h \Delta x, dA_h = A_h \frac{\Delta x}{L}, A_h = P_h L \text{ \& } P_h = \pi(D + 2t_w) \dots\dots\dots (16)$$

Where “ t_w ” is too small, the conduction resistance inside the pipe material is low compared with the convection resistance, then, we neglected the conduction thermal resistance inside the pipe material compared with the convection resistance.

By applying the energy balance for the cold fluid, we get

$$\sum Q_{in} - \sum Q_{out} = \text{Change in Internal Energy} \dots\dots\dots (17)$$

$$Q_{c,i} + Q_{w,c} - Q_{c,o} = Q_c \dots\dots\dots (18)$$

$$Q_{c,i} + Q_{w,c} - Q_{c,i} - \frac{\partial Q_{c,i}}{\partial x} \Delta x = Q_c \dots\dots\dots (19)$$

$$h_c (T_w - T_c) P_c \Delta x - \frac{\partial(\dot{m}_c C p_c T_c)}{\partial x} \Delta x = \rho_c V_c \frac{\Delta x}{L} C p_c \frac{\partial T_c}{\partial t} \dots\dots\dots (20)$$

By substitute the eq. (2) in eq. (20).

$$\rho_c V_c C_p \frac{\partial T_c}{\partial t} = h_c (T_w - T_c) P_c L - \frac{\partial(\dot{m}_c C_{p_{oc}}(1 + \beta T_c))}{\partial x} L \quad (21)$$

$$\rho_c V_c C_{p_{oc}}(1 + \beta T_c) \frac{\partial T_c}{\partial t} = h_c (T_w - T_c) A_c - \dot{m}_c C_{p_{oc}} L [(1 + \beta T_c) + \beta T_c] \frac{\partial T_c}{\partial x} \quad (22)$$

Finally

$$(1 + \beta T_c) \frac{\partial T_c}{\partial t} = \frac{h_c A_c}{\rho_c V_c C_{p_{oc}}} (T_w - T_c) - \frac{\dot{m}_c C_{p_{oc}}}{\rho_c V_c C_{p_{oc}}} L [1 + 2\beta T_c] \frac{\partial T_c}{\partial x} \quad (23)$$

For the hot fluid,

$$(1 + \beta T_h) \frac{\partial T_h}{\partial t} = \frac{h_h A_h}{\rho_h V_h C_{p_{oh}}} (T_w - T_h) + \frac{\dot{m}_h C_{p_{oh}}}{\rho_h V_h C_{p_{oh}}} L [1 + 2\beta T_h] \frac{\partial T_h}{\partial x} \quad (24)$$

For the pipe material,

$$(1 + \beta T_w) \frac{\partial T_w}{\partial t} = \frac{h_h A_h}{\rho_w V_w C_{p_{ow}}} (T_h - T_w) + \frac{h_c A_c}{\rho_w V_w C_{p_{ow}}} (T_c - T_w) \quad (25)$$

For dimensionless analysis

$$\theta_{c,h,w}(x, t) = \frac{T_{c,h,w}(x, t) - T_{c,i}}{T_{h,i} - T_{c,i}} \quad (26)$$

$$C_{p_{c,h,w}} = C_{p_{oc,h,w}} (1 + \alpha \theta_{c,h,w}) \quad (27)$$

$$X' = \frac{x}{L}, \tau = \frac{t}{\tau_o}, N_{c,h} = \frac{h_{c,h} A_{c,h}}{\dot{m}_{c,h} C_{p_{oc,h}}}, \tau_o = \frac{\rho_c V_c C_{p_{oc}}}{\dot{m}_c C_{p_{oc}}}, C_r = \frac{\dot{m}_h C_{p_{oh}}}{\dot{m}_c C_{p_{oc}}}$$

$$\text{and } E_{c,h,w} = \frac{\rho_{c,h,w} V_{c,h,w} C_{p_{oc,h,w}}}{\rho_c V_c C_{p_{oc}}}$$

Then the final dimensionless form are:

For cold fluid,

$$\frac{\partial \theta_c}{\partial \tau} = \frac{1}{(1 + \alpha \theta_c)} [N_c (\theta_w - \theta_c) - (1 + 2\alpha \theta_c) \frac{\partial \theta_c}{\partial X'}] \quad (28)$$

For the hot fluid

$$\frac{\partial \theta_h}{\partial \tau} = \frac{C_r}{E_h (1 + \alpha \theta_h)} [N_h (\theta_w - \theta_h) + (1 + 2\alpha \theta_h) \frac{\partial \theta_h}{\partial X'}] \quad (29)$$

For the pipe material

$$\frac{\partial \theta_w}{\partial \tau} = \frac{N_c}{E_w (1 + \alpha \theta_w)} \left[\frac{N_h C_r}{N_c} (\theta_h - \theta_w) + (\theta_c - \theta_w) \right] \quad (30)$$

Solution Analysis

In our steady, we used finite difference techniques for solving the above differential equations (28),(29),(30) :

$$\frac{\theta_{c,m}^{n+1} - \theta_{c,m}^n}{\Delta \tau} = \frac{1}{(1 + \alpha \theta_{c,m}^n)} \left[N_c (\theta_{w,m}^n - \theta_{c,m}^n) - (1 + 2\alpha \theta_{c,m}^n) \left(\frac{\theta_{c,m}^n - \theta_{c,m-1}^n}{\Delta X'} \right) \right] \quad (31)$$

$$\frac{\theta_{h,m}^{n+1} - \theta_{h,m}^n}{\Delta \tau} = \frac{C_r}{E_h (1 + \alpha \theta_{h,m}^n)} \left[N_h (\theta_{w,m}^n - \theta_{h,m}^n) - (1 + 2\alpha \theta_{h,m}^n) \left(\frac{\theta_{h,m+1}^n - \theta_{h,m}^n}{\Delta X'} \right) \right] \quad (32)$$

$$\frac{\theta_{w,m}^{n+1} - \theta_{w,m}^n}{\Delta\tau} = \frac{N_c}{(1 + \alpha\theta_{w,m}^n)} \left[\frac{N_h C_r}{N_h} (\theta_{h,m}^n - \theta_{w,m}^n) - (\theta_{c,m}^n - \theta_{w,m}^n) \right] \dots\dots\dots (33)$$

For more simplifications, we get:

$$\theta_{c,m}^{n+1} = \frac{\Delta\tau}{\Delta X'(1 + \alpha\theta_{c,m}^n)} \left[N_c \Delta X' \theta_{w,m}^n + (1 + 2\alpha\theta_{c,m}^n) \theta_{c,m-1}^n + \left(\frac{\Delta X'(1 + \alpha\theta_{c,m}^n)}{\Delta\tau} - (1 + 2\alpha\theta_{c,m}^n) - N_c \Delta X' \right) \theta_{c,m}^n \right] \dots\dots\dots (34)$$

$$\theta_{h,m}^{n+1} = \frac{\Delta\tau C_r}{\Delta X' E_h (1 + \alpha\theta_{h,m}^n)} \left[N_h \Delta X' \theta_{w,m}^n + (1 + 2\alpha\theta_{h,m}^n) \theta_{h,m+1}^n + \left(\frac{\Delta X' E_h (1 + \alpha\theta_{h,m}^n)}{\Delta\tau C_r} - N_h \Delta X' - (1 + 2\alpha\theta_{h,m}^n) \right) \theta_{h,m}^n \right] \dots\dots\dots (35)$$

$$\theta_{w,m}^{n+1} = \frac{N_c \Delta\tau}{E_w (1 + \alpha\theta_{w,m}^n)} \left[\frac{N_h C_r}{N_c} \theta_{h,m}^n + \theta_{c,m}^n + \left(\frac{E_w (1 + \alpha\theta_{w,m}^n)}{N_c \Delta\tau} - 1 - \frac{N_h C_r}{N_c} \right) \theta_{w,m}^n \right] \dots\dots\dots (36)$$

The best (Δt) which used in the present analysis is selected from the above equations, which made all the above equations is positive ,(then we will select the lower value of the (Δt)),as following;

The best (Δt) for the cold fluid can written as

$$\Delta t_c = \frac{(1 + \alpha\theta_c) \Delta X'}{N_c \Delta X' + (1 + 2\alpha\theta_c)} \dots\dots\dots (37)$$

The best (Δt) for the hot fluid can written as

$$\Delta t_h = \frac{E_h \Delta X' (1 + \alpha\theta_h)}{C_r [N_h \Delta X' + (1 + 2\alpha\theta_h)]} \dots\dots\dots (38)$$

The best (Δt) for the pipe wall can written as

$$\Delta t_w = \frac{E_w (1 + 2\alpha\theta_w)}{N_h C_r + N_c} \dots\dots\dots (39)$$

Heat Transfer Coefficient

Many correlation exist in literature for circular & annulus area,[9,10,11,12]. In the present study, The calculation of the heat transfer coefficient for the double pipe heat exchanger fluids are following:

For the hot fluid, the heat transfer coefficient for the fluid inside the tube,[8], can determined in eq.(40), this equation is widely used for the flow inside the tube,[9,10,11,12].

$$Nu_h = 0.023 Re_h^{0.8} Pr_h^{0.3} \dots\dots\dots (40)$$

For the cold fluid, the heat transfer coefficient for the fluid inside the annulus tube,[8], can determined in eq.(41),this equations proposed by Kawamura, [],for turbulent flow. This equation is depend also on the diameter ratio between the outer to inner radii. Bagui,[7], shows this formula is accurate & he compared it with experimental results.

$$Nu_c = 0.022 \phi_i Re_h^{0.8} Pr_h^{0.5} \dots\dots\dots (41)$$

$$\phi_i = \frac{r_m^2 - 1}{r_c^2 - 1} \dots\dots\dots (42)$$

$$r_c = \frac{r_o}{r_i} \dots\dots\dots (43)$$

$$r_m = r_{m\infty} + (r_{ml} - r_{m\infty}) \left(\frac{2300}{Re_h} \right)^{0.8} \dots\dots\dots (44)$$

$$r_{ml} = \sqrt{\frac{r_c^2 - 1}{2 \ln(r_c)}} \dots\dots\dots (45)$$

$$r_{m\infty} = \frac{r_c^{0.415} + r_c}{r_c^{0.415} + 1} \dots\dots\dots (46)$$

$$Re = \frac{4 \dot{m}}{\pi D \mu} \dots\dots\dots (47)$$

Heat Exchanger Effectiveness (ϵ)

Heat exchanger effectiveness, ϵ , is defined as the ratio of the actual heat transfer rate (Q_{actal}) to the thermodynamically possible maximum heat-transfer rate (Q_{max}) by the second law of thermodynamics, the value of the effectiveness ranges between 0 and 1.

$$\epsilon = \frac{Q_{actual}}{Q_{max}} \dots\dots\dots(48)$$

The maximum possible heat transfer rate would be obtained in a counter flow heat exchanger with very large surface area and zero longitudinal wall heat conduction, and the actual operating conditions are same as the theoretical conditions.

Because of not all energy that removed from the hot fluid will transfer to the cold fluid, there, we will discussed three types of efficiency, the first type for the cold fluid, second type for the hot fluid, the third for the energy transfer between hot to cold fluid at instantaneous point.

First type, for the hot fluid :

$$\epsilon_h = \frac{Q_h}{Q_{max,h}} \dots\dots\dots (49)$$

$$Q_h = \dot{m}_h C_{p,h} (T_{h,i} - T_{h,o}) \dots\dots\dots (50)$$

$$Q_{max,h} = \dot{m}_h C_{p,h} (T_{h,i} - T_{C,i}) \dots\dots\dots (51)$$

$$\text{Then } \epsilon_h = \frac{(T_{h,i} - T_{h,o})}{(T_{h,i} - T_{C,i})} \dots\dots\dots (52)$$

Second type, for the cold fluid :

$$\epsilon_c = \frac{Q_c}{Q_{max,c}} \dots\dots\dots (53)$$

$$Q_c = \dot{m}_c C_{p,c} (T_{c,o} - T_{c,i}) \dots\dots\dots (54)$$

$$Q_{max,c} = \dot{m}_c C_{p,c} (T_{h,i} - T_{C,i}) \dots\dots\dots (55)$$

$$\text{Then } \epsilon_c = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{C,i})} \dots\dots\dots (56)$$

Third type, here will called instantaneous efficiency:

$$\epsilon_a = \frac{\text{Instantaneous actual heat transfer from the hot fluid to the cold fluid } (Q_a)}{\text{Maximum possible heat transfer rate would be obtained } (Q_{max})} \dots\dots\dots (57)$$

$$Q_a = UA(T_h - T_C) \dots\dots\dots (58)$$

Where

$$\frac{1}{UA} = \frac{1}{h_h A_h} + \frac{1}{h_c A_c} \dots\dots\dots (59)$$

$$Q_{\max} = C_{\min} (T_{h,i} - T_{c,i}) \dots\dots\dots (60)$$

Where C is the product of the mass flow rate and the specific heat, as a matter of convenience, is defined as the fluid capacity rate.

$$C_h = \dot{m}_h C_{p_h}, C_c = \dot{m}_c C_{p_c} \dots\dots\dots (61)$$

$$C_{\min} = C_h \text{ or } C_c \text{ (minimum fluid capacity rate)}$$

The Entropy Generation Number (Ns)

The entropy increase of the universe as the result of a process is the sum of the entropy changes of all elements that are involved in that process. There are two sources of entropy generation in heat exchangers. One is due to heat exchange between two fluid streams of different temperature, and the second one due to viscosity (frictional pressure drop) of moving fluids. The entropy generation rate due to viscosity is usually much smaller than the generation due to heat exchange, so it can be neglected,[13]. These losses refer to irreversibility quantity, and some methods have been devised for minimizing these losses, [13].

The rate of entropy production of the universe,

$$\dot{S}_{\text{gen}} = \dot{S}_{\text{genh}} + \dot{S}_{\text{genc}} + \dot{S}_{\text{genw}} \dots\dots\dots (62)$$

Where $\dot{S}_{\text{genw}} = 0$ since these term is constant with time.

Since the entropy generation according to the second law of thermodynamic can expressed as following:

$$\dot{S}_{\text{gen}} = \frac{Q_{\text{gen}}}{T_{\text{rev}}} \dots\dots\dots (63)$$

Then, in the analyzing of the heat exchangers must be determined the heat transfer for each term are:

$$Q_h = - \int_{A_h} U(T_h - T_c) dA \dots\dots\dots (64)$$

$$Q_c = \int_{A_c} U(T_h - T_c) dA \dots\dots\dots (65)$$

The entropy generation rate due to heat transfer between the heat exchanger working fluid in eq.(37) can be defined as,[14]:

$$\dot{S}_{\text{Un}} = - \int_{A_h} \frac{Q_h}{T_h} dA + \int_{A_c} \frac{Q_c}{T_c} dA = - \int_{A_h} \frac{U(T_h - T_c)}{T_h} dA + \int_{A_c} \frac{U(T_h - T_c)}{T_c} dA \dots\dots\dots (66)$$

If we assumed the $A_h = A_c$, then the eq.(66) can be written as:

$$\dot{S}_{\text{Un}} = U \int_{A_h} \frac{(T_h - T_c)}{T_h T_c} dA \dots\dots\dots (67)$$

Because of the $T_h > T_c$, the value of entropy generation in eq.(67) is always greater than zero (positive).

Bejan,[13], used a parameter called entropy generation number for minimizing both losses, and described this parameter as the ratio between entropy generation rate and the overall heat transfer coefficient. The entropy generation number limit $N_s \rightarrow 0$ implies that these losses approach zero,

and that these losses increase when N_s has higher values. Entropy generation can be written as, [14].

$$N_s = \frac{\dot{S}_{Un}}{UA} = \frac{1}{A_h A_h} \int \frac{(T_h - T_c)}{T_h T_c} dA \quad \dots\dots\dots (70)$$

Case Study

In our case study, The information of the countercurrent double pipe heat exchanger,[7], as mentioned below:

The information of the heat exchanger tubes			the information of the heat exchanger fluids		
Parameter	Inner Tube	Outer Tube	Parameter	Cold (Water)	Hot (Oil)
k (W/m.°C)	384	45	ṁ (Kg/s)	0.3	0.5
Cp (J/Kg.°C)	394	490	ρ (Kg/m³)	1000	800
ρ (Kg/m³)	8900	7850	Cp (J/Kg.°C)	4200	1900
D (m)	2×10 ⁻²	4×10 ⁻²	v (m²/Sec)	7×10 ⁻⁷	1×10 ⁻⁵
t (m)	10 ⁻³	3×10 ⁻³	k (W/m.°C)	0.64	0.134
L (m)	4.5	4.5	Pr	4.7	140
			α (1/°C)	0.1	0.1
			Ti (°C)	30	100

Results & Discussion

After writing a computer program to solve the above differential equations. The results of the computer program are plotted here. In figure (2), we plotted the dimensionless temperatures of the hot, cold fluids & the pipe material along the dimensionless of heat exchanger length. We can notes the temperature of the cold fluid is increase with increase of the heat exchanger length & in the same time the temperature of the hot fluid is decreased because some energy is transfer from the hot fluid to the cold fluid (the thermal capacitance of the hot fluid decreased with time while the thermal capacitance of the cold energy increased with time) & this energy is transfer through the pipe wall, therefore we can notes the temperature of the pipe is increased along the heat exchanger length.

The variation of the heat exchanger effectiveness for cold, hot fluids & the instantaneous effectiveness are plotted against the dimensionless time in figure (3), we can notes the hot fluid efficiency decrease as dimensionless time increased because of the temperature of the hot fluid temperature is decrease with time (due to reduce of the hot fluid internal energy) & also we can notes the effectiveness of the cold fluid will increase with time due to the temperature of the cold fluid is increased with time (due to increase of the hot fluid internal energy). We can also notes the instantaneous effectiveness decreased as time increase, because this effectiveness is depends on the temperature difference between the hot & cold fluid & this difference is reduced as the time increase, then the instantaneous effectiveness decreased as the time increased.

The variation of temperature profile for the hot fluid against dimensionless heat exchanger length for different values of dimensionless time are plotted in figure (4), as the time increase, the temperature of the hot fluid is decreased due to decreasing of the internal energy (increase the heat removed from the hot fluid). The difference between temperature profile lines will decrease as the time increase due to decreasing of the heat removed (internal energy) from the hot fluid as time increase.

The effect of the (C_r) on the hot fluid effectiveness are graphed in figure (5) for different values of the dimensionless temperature, the increasing of the (C_r) will increase the hot fluid efficiency because will increase the mass flow rate (heat capacitance) of the hot fluid compared with the

heat capacitance of the cold fluid & then increase hot fluid effectiveness due to decrease the temperature profile line slope(internal energy) for the hot fluid.

The hot fluid effectiveness are plotted with different values of (N_c) against dimensionless time in figure(6). The increasing of the (N_c) values will increase the hot fluid efficiency by decreasing the mass capacity (internal energy) of the cold fluid, then the energy transfer from the hot fluid to cold fluid will reduce , then hot fluid temperature line slope is decreased (the heat removed from the internal energy is not more). For the same previous reasons, the effectiveness of the cold fluid is decreased with increasing of (N_c) values due to decreasing the internal energy (thermal capacitance) of the cold fluid (see figure (7)).

The effect of the (N_h) on the hot fluid effectiveness & cold fluid efficiency are plotted in figure (8) & figure (9). The increasing of the (N_h) will increase the hot fluid efficiency & decreasing the cold fluid efficiency.

The effect of the (E_w) on hot fluid effectiveness are plotted in figure (10), at (E_w) values less than (1), the heat capacitance for the cold fluid is greater than the heat capacitance for the hot fluid, then the efficiency of the hot fluid is decrease as the time increase due to reducing in the heat capacitance of the hot fluid (the heat removed from the hot fluid to the cold fluid increase due to the increasing of the cold fluid capacity), but when the values of (E_w) become greater than (1), the efficiency of the hot fluid will increase (the thermal capacitance increase), then the energy transfer from the hot fluid to the cold fluid decreased (the cold fluid capacitance is small). At the time nearly zero, the effect of the (E_h) is not noticeable & the increasing of the time will become the (E_h) more effective due to the cold & hot fluids have initial values at time near zero.

The effect of the Dimensionless constant for heat capacity coefficient (α) is plotted in figure (11), the increasing of (α) will increase the hot fluid effectiveness because increase of (α) will increase the internal energy for the fluid and then increase the temperature of the fluid.

Finally we will study the effect of the other parameters on the entropy generation. The variation of the entropy generation number with dimensionless time is plotted in figure (12), the entropy generation is reduce as the time increase because the heat transfer rate as the time decrease.

The increasing of the (C_r) will decrease the (N_s) due to increase of the C_r will decrease the number of entropy generation due to increase of C_r will decrease the temperature difference between the hot & cold fluid.

Conclusions

The unsteady state countercurrent double pipe heat exchanger differential equations is solved a numerically by using finite difference technique with variable specific heat coefficient for hot, cold fluid and tube wall. The effect of the time, N_c , N_h , E_h , C_r on the cold & hot fluid effectiveness are and also generation number is presented. The increasing of the C_r will increase the hot fluid effectiveness and the entropy generation number, the increasing of the N_c & N_h values will increase the hot fluid effectiveness & in same time decrease the cold fluid effectiveness & also the increasing of the C_r will increase the hot fluid effectiveness and the entropy generation number. Our analysis is considering hot fluid (that we need to cool it) flow inside the tube & the cold fluid flows inside the annulus area.

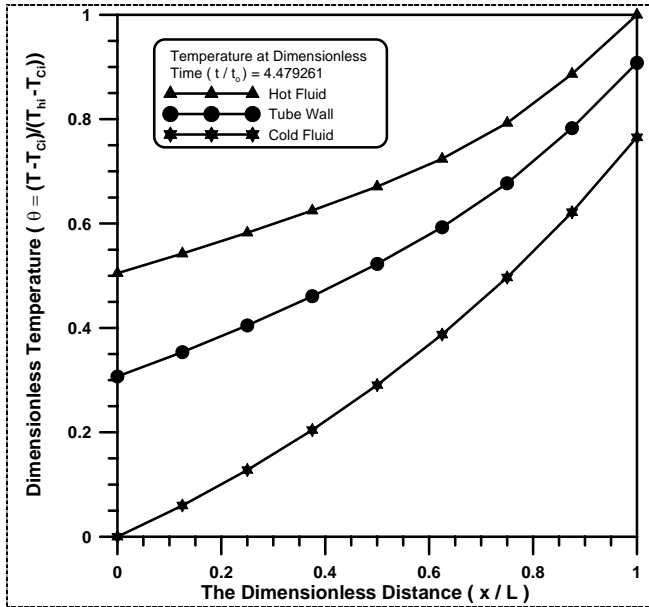


Fig.(2): Dimensionless temperature along the heat exchanger length

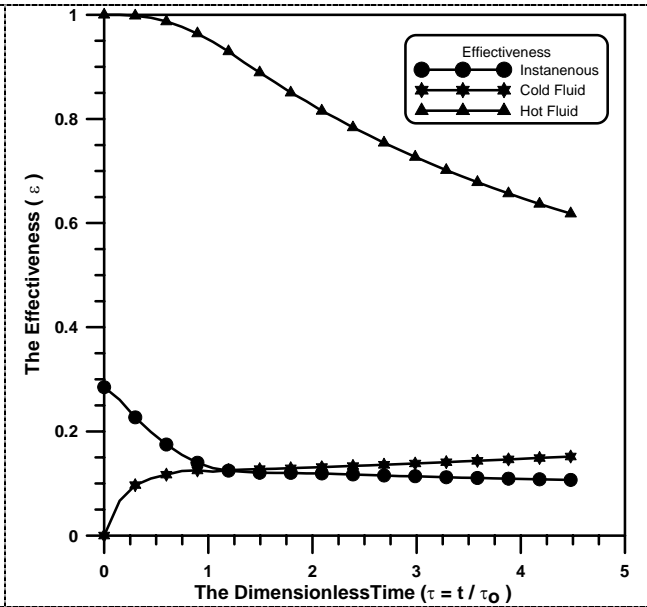


Fig.(3): The variation of heat exchanger effectiveness with dimensionless time

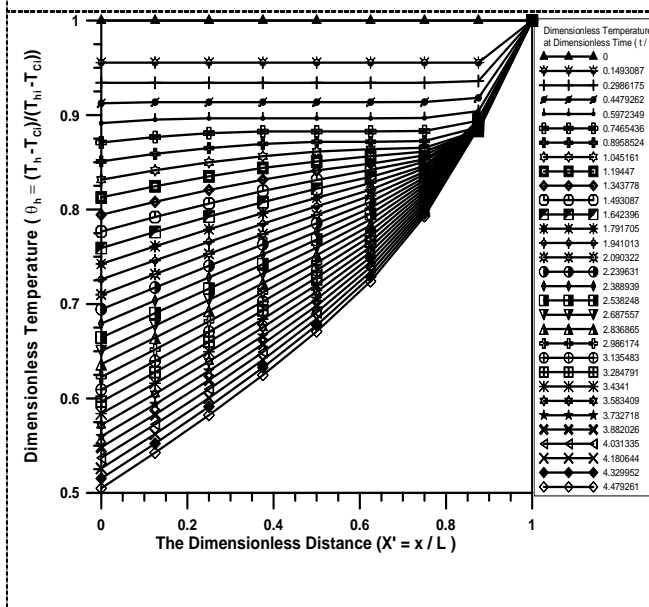


Fig.(4): Dimensionless temperature of hot fluid along the heat exchanger length

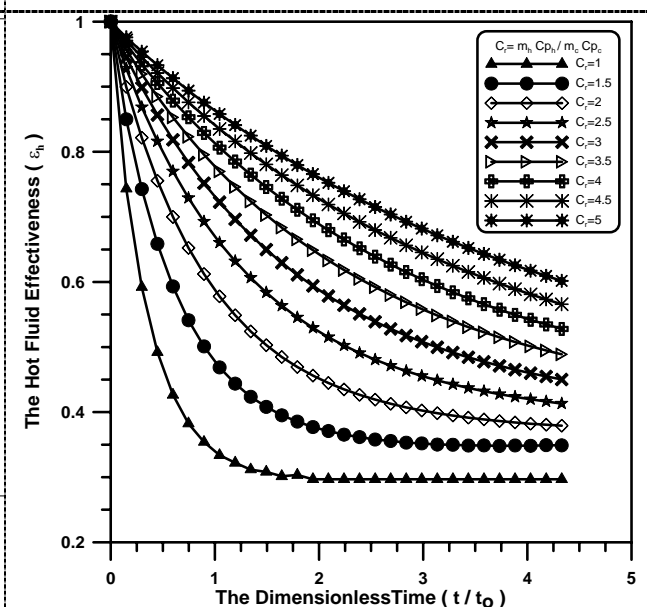


Fig.(5): The variation of hot fluid heat exchanger effectiveness with dimensionless time for different (C_r)

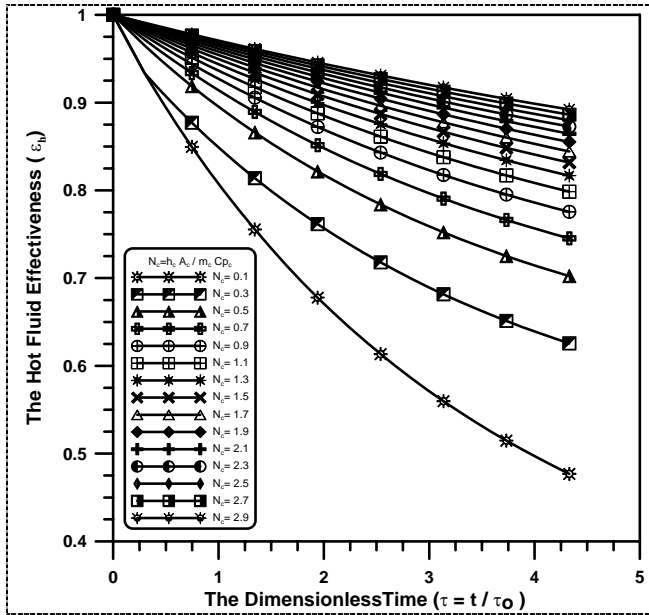


Fig.(6): The variation of hot fluid heat exchanger effectiveness with dimensionless time for different (N_c)

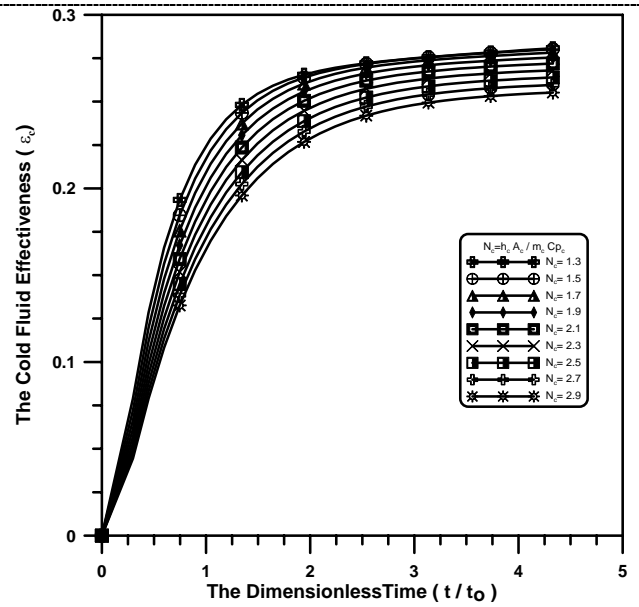


Fig.(7): The variation of cold fluid heat exchanger effectiveness with dimensionless time for different (N_c)

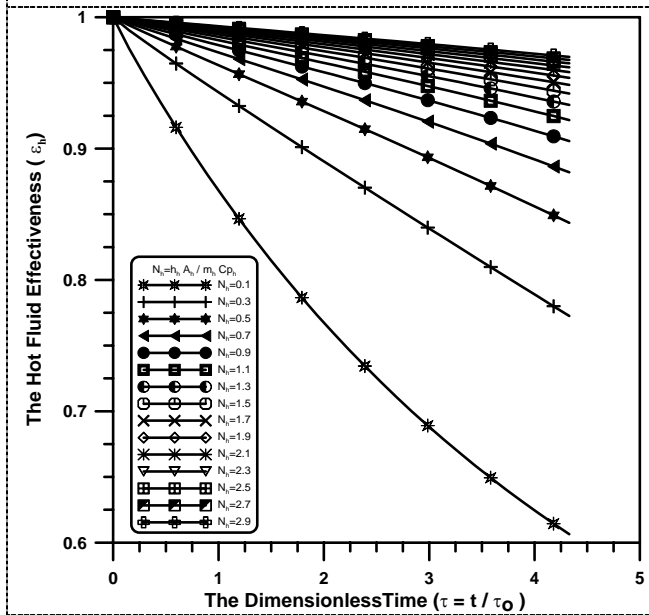


Fig.(8): The variation of hot fluid heat exchanger effectiveness with dimensionless time for different (N_h)

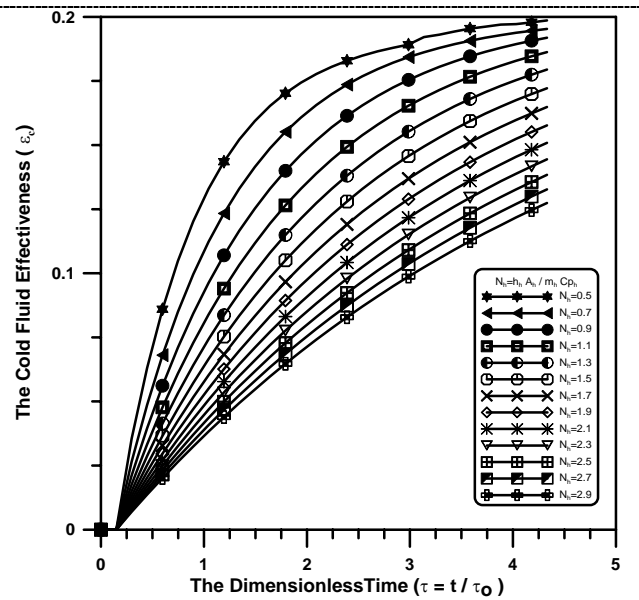


Fig.(9): The variation of cold fluid heat exchanger effectiveness with dimensionless time for different (N_h)

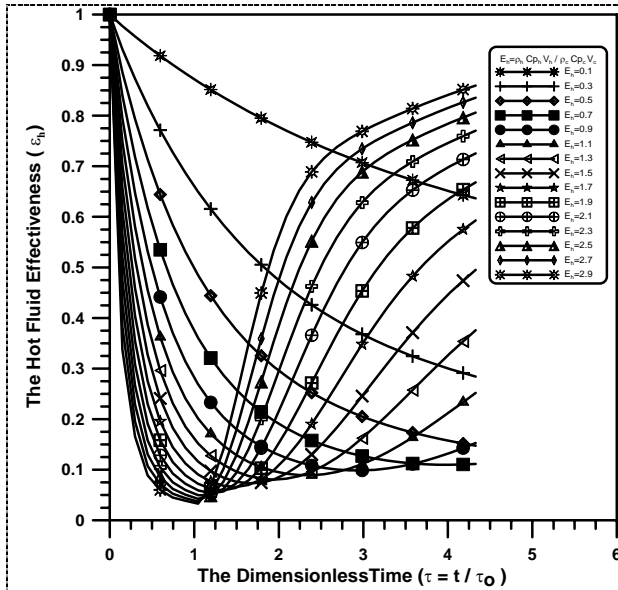


Fig.(10): The variation of hot fluid heat exchanger effectiveness with dimensionless time for different (E_h)

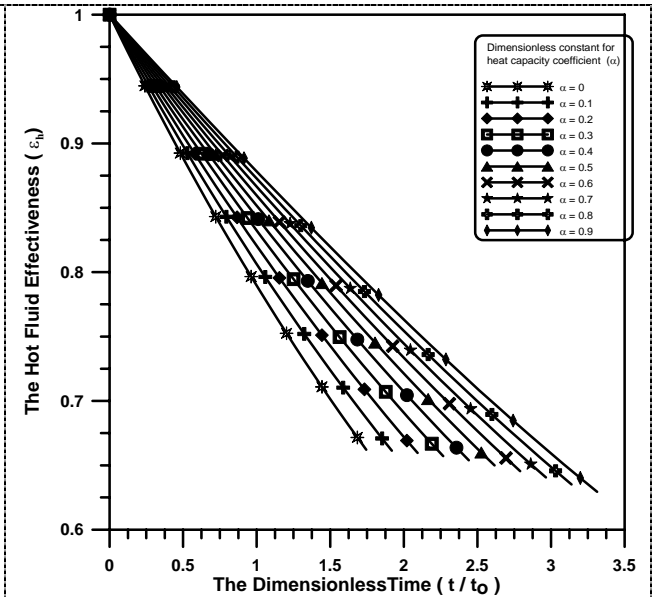


Fig.(11): The variation of hot fluid heat exchanger effectiveness with dimensionless time for different (α)

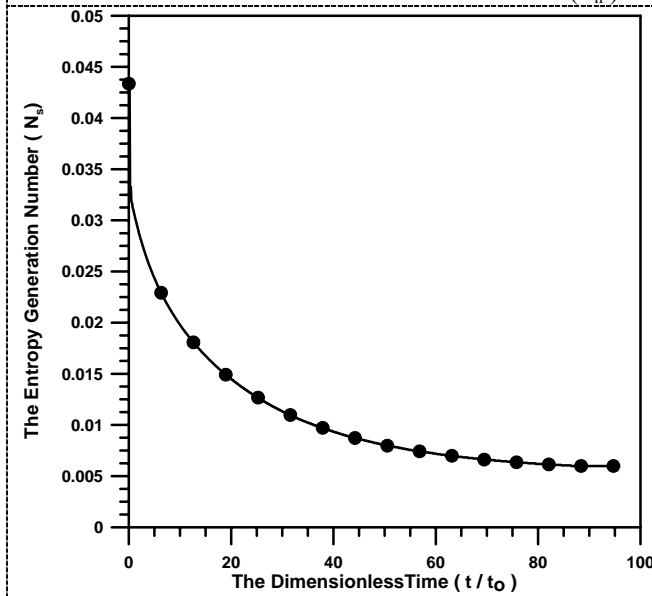


Fig.(12): The variation of entropy generation number effectiveness with dimensionless time

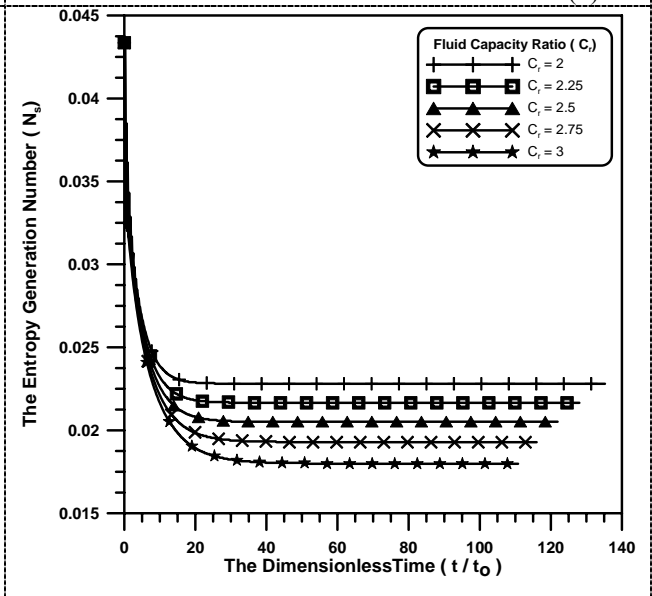


Fig.(13): The variation of entropy generation number effectiveness with dimensionless time for different (C_r)

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